

Brane inflation: A field theory approach in background supergravity

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Abstract. We propose a model of inflation in the framework of brane cosmology driven by background supergravity. Starting from bulk supergravity we construct the inflaton potential on the brane and employ it to investigate for the consequences to inflationary paradigm. To this end, we derive the expressions for the important parameters in brane inflation, which are somewhat different from their counterparts in standard cosmology, using the one loop radiative corrected potential. We further estimate the observable parameters and find them to fit well with recent observational data. We have studied extensively reheating phenomenology, which explains the thermal history of the universe and leptogenesis through the production of thermal gravitino pertaining to the particle physics phenomenology of the early universe.

1. Introduction

Investigations for the crucial role of Supergravity in explaining cosmological inflation date back to early eighties of the last century [1],[2]. One of the generic features of the inflationary paradigm based on SUGRA is the well-known η -problem, which appears in the F-term inflation due to the fact that the energy scale of F-term inflation is induced by all the couplings via vacuum energy density. Precisely, in the expression of F-term inflationary potential a factor $\exp(K/M_{PL})$ appears, leading to the second slow roll parameter $\eta \gg 1$, thereby violating an essential condition for slow roll inflation. The usual way out is to impose additional symmetry like Nambu-Goldstone shift symmetry or alternatively to apply Heisenberg symmetry [3] to solve η -problem. Another possible way out to smoothen this problem is fine tuning mechanism via the *radion* fields –which decides the separation between visible and invisible brane or so as to say brane tension in Randall-Sundrum two-brane scenario where the fifth dimension of 5 D bulk is compactified on the orbifold S^1/Z_2 of comoving radius R . This is essential for observationally constraint cosmology on the brane. As we will find in the present article the proposed model of brane inflation matches quite well with latest observational data from WMAP [4] and is expected to fit well with upcoming data from Planck [5]. Next we have studied in detail reheating and leptogenesis, which is different in the context of brane inflation results in novel features. Hence we have established that this has serious implication for the production of the heavy Majorana neutrinos needed for leptogenesis as well as in the gravitino phenomenology through analytical and numerical estimations.

2. Model building from background supergravity

For systematic development of the formalism, let us briefly demonstrate the construction the effective 4D inflationary potential starting from $N = 2, D = 5$ SUGRA in the bulk which leads to an effective $N = 1, D = 4$ SUGRA in the brane [3]. Considering S^1/Z_2 orbifold compactification in comoving radius R , the $N = 2, D = 5$ bulk SUGRA is described by the following action

$$S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{g_5} \left[M_5^3 (R_{(5)} - 2\Lambda_5) + L_{SUGRA}^{(5)} + \sum_{i=1}^2 \delta(y - y_i) L_{4i} \right]. \quad (1)$$

Here the orbifold points are $y_i = (0, \pi R)$ and $x^m = (x^\alpha, y)$, where y is bounded in closed interval $[-\pi R, +\pi R]$. Written explicitly, the contribution from bulk SUGRA in the action

$$e_{(5)}^{-1} L_{SUGRA}^{(5)} = -\frac{M_5^3 R^{(5)}}{2} + \frac{i}{2} \bar{\Psi}_{i\tilde{m}} \Gamma^{\tilde{m}\tilde{n}\tilde{q}} \nabla_{\tilde{n}} \Psi_{\tilde{q}}^i - S_{IJ} F_{\tilde{m}\tilde{n}}^I F^{I\tilde{m}\tilde{n}} - \frac{1}{2} g_{\alpha\beta} (D_{\tilde{m}} \phi^\mu) (D^{\tilde{m}} \phi^\nu) + \text{Fermionic} + \text{Chern - Simons}, \quad (2)$$

and including the radion fields (χ, T, T^\dagger) the effective brane SUGRA counterpart turns out to be $\delta(y) L_4 = -e_{(5)} \Delta(y) \left[(\partial_\alpha \phi)^\dagger (\partial^\alpha \phi) + i \bar{\chi} \bar{\sigma}^\alpha D_\alpha \chi \right]$. Gauging away the Chern-Simons terms assuming cubic constraints and Z_2 symmetry and applying S^1/Z_2 orbifold compactification the effective 4-dimensional action can be expressed as,

$$S = \frac{M_{PL}^2}{2} \int d^4x \sqrt{g_4} \left[R_{(4)} + (\partial_\alpha \phi^\mu)^\dagger (\partial^\alpha \phi_\mu) - QV_F - P \int_{-\pi R}^{+\pi R} dy \frac{4(3e^{2\beta y} + 3\lambda^2 e^{-2\beta y} - 2\lambda)}{R^2(e^{\beta y} + \lambda e^{-\beta y})^5} \right]. \quad (3)$$

where the constants are defined in [3]. Now using the Kähler potential $K = \sum_\alpha \phi_\alpha^\dagger \phi^\alpha$ and the superpotential $W = \sum_{n=0}^\infty D_n W_n(\phi^\alpha)$ with $D_0 = 1$ and Z_2 symmetry the renomalizable effective one-loop F-term potential $N = 1, D = 4$ SUGRA in the brane turns out to be [3]

$$V(\phi) = \Delta^4 \left[1 + \left(D_4 + K_4 \ln \left(\frac{\phi}{M} \right) \right) \left(\frac{\phi}{M} \right)^4 \right], \quad (4)$$

where K_4 and D_4 are one loop level constants. For our model energy scale $\Delta \simeq 0.2 \times 10^{16} \text{ GeV}$ for the window $-0.70 < D_4 < -0.60$.

3. Brane inflation and observational parameters

Here we start with Friedmann equations in brane $H^2 = \frac{8\pi V}{3M_{PL}^2} \left[1 + \frac{V}{2\lambda} \right]$. Incorporating the potential of our consideration from Eq (4) the new slow roll parameters turn out to be [6]

$$\epsilon_V = \left(\frac{V'}{V} \right)^2 \frac{M_{PL}^2 (1 + \frac{V}{2\lambda})}{16\pi (1 + \frac{V}{2\lambda})^2}, \eta_V = \left(\frac{V''}{V} \right) \frac{M_{PL}^2}{8\pi (1 + \frac{V}{2\lambda})}, \xi_V = \left(\frac{V' V'''}{V^2} \right) \frac{M_{PL}^4}{(8\pi)^2 (1 + \frac{V}{2\lambda})^2}, \sigma_V = \frac{(V')^2 V''''}{V^3} \frac{M_{PL}^6}{(8\pi)^3 (1 + \frac{V}{2\lambda})^3}, \quad (5)$$

Most significantly the generic SUGRA η -problem is smoothened to some extent in brane cosmology due to the presence of V/λ term in Friedmann equation. The number of e-foldings are defined in brane cosmology for our model as

$$N \simeq \frac{8\pi}{M_{PL}^2} \int_{\phi_f}^{\phi_i} \left(\frac{V}{V'} \right) \left(1 + \frac{V}{2\lambda} \right) d\phi \simeq \frac{M^2}{2U} \left[\left(1 + \frac{\alpha}{2} \right) \left(\frac{1}{\phi_f^2} - \frac{1}{\phi_i^2} \right) + \frac{D_4}{M^4} (1 + \alpha) (\phi_i^2 - \phi_f^2) + \frac{\alpha D_4^2}{6M^8} (\phi_i^6 - \phi_f^6) \right], \quad (6)$$

where $U = (D_4 + 4K_4)$. After that the expressions for amplitude of the scalar and tensor perturbation and tensor to scalar ratio [6] are given by

$$\Delta_s^2 \simeq \frac{512\pi}{75M_{PL}^6} \left[\frac{V^3}{(V')^2} \left[1 + \frac{V}{2\lambda} \right]^3 \right]_\star, \Delta_t^2 \simeq \left[\frac{\frac{32}{75M_{PL}^4} V \left[1 + \frac{V}{2\lambda} \right]}{\sqrt{1 + \frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right)} - \frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right) \sinh^{-1} \left[\frac{1}{\sqrt{\frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda} \right)}} \right]} \right]_\star, r = 16 \frac{\Delta_t^2}{\Delta_s^2}. \quad (7)$$

Here \star represents at the horizon crossing ($k = aH$). Further, the scale dependence of the perturbations, described by the scalar and tensor spectral indices and their running as follows

$$\begin{aligned} n_s - 1 &= \frac{d \ln \Delta_s^2}{d \ln k} \Big|_{\star} \simeq (2\eta_V^{\star} - 6\epsilon_V^{\star}), & n_t &= \frac{d \ln \Delta_t^2}{d \ln k} \Big|_{\star} \simeq -3\epsilon_V^{\star}, \\ \alpha_s &= \frac{d \ln n_s}{d \ln k} \Big|_{\star} = (16\eta^{\star}\epsilon^{\star} - 18(\epsilon^{\star})^2 - 2\xi^{\star}), & \alpha_t &= \frac{d \ln n_t}{d \ln k} \Big|_{\star} = (6\epsilon^{\star}\eta^{\star} - 9(\epsilon^{\star})^2), \end{aligned} \quad (8)$$

where $d(\ln(k)) = H dt$. Here the consistency conditions $r = 24\epsilon_V = 24\epsilon_V^{\star}$; $n_t = -3\epsilon_V \simeq -3\epsilon_V^{\star} = -\frac{r}{8}$ and $\frac{d\sigma}{d(\ln(k))} = (\epsilon\sigma - 2\eta\sigma)$ are valid in brane cosmology.

4. Reheating and Leptogenesis in braneworld

After inflation when reheating epoch starts inflaton decays into different particle constituents. The total inflaton decay width participating in leptogenesis can be written as [7]:

$$\Gamma_{total}(\psi \rightarrow l_L \mathcal{H}, \psi \rightarrow \bar{l}_L \mathcal{H}) \simeq \frac{C^2}{16\pi m_{\phi}} + \frac{h^2 m_{\psi}}{4\pi} \sim \frac{1}{(2\pi)^3} \left(\frac{\Delta^6}{M^5} \right) = 3H(T^{brh}) = \sqrt{\frac{3\rho(t_{reh})}{M^2} \left[1 + \frac{\rho(t_{reh})}{2\lambda} \right]}. \quad (9)$$

Now applying statistical thermodynamics in braneworld the expression for transition temperature($T_{c\gamma}$) and the reheating temperature(T^{brh}) turns out to be

$$T_{c\gamma} = \sqrt[4]{\left\{ C_{\gamma} \frac{\pi^2 (K_4 + 4D_4)^2 \Delta_s^2 \phi_s^6}{\pi^2 N_{\gamma}^* \alpha^4 M^2} \right\}}, \quad T_{\gamma}^{brh} = \sqrt[4]{\left\{ \frac{W_{\gamma} (K_4 + 4D_4) \Delta_s \phi_s^3 \Gamma_{total}}{\pi N_{\gamma}^* \alpha^2} \right\}} \quad \forall \gamma \quad (10)$$

where $C_{\gamma} = \left(36000, \frac{288000}{7}, 19200 \right)$, $W_{\gamma} = \left(600, \frac{4800}{7}, 320 \right)$, the species index $\gamma = 1(B \Rightarrow Boson), 2(F \Rightarrow Fermion), 3(M \Rightarrow Mixture)$ and $N^* = N_B^* + \frac{7}{8}N_F^*$.

To study the impact of ϕ^4 in leptogenesis we will start with the Boltzmann equation [8]

$$\dot{\rho}_r + 4H\rho_r = \Gamma_{\phi}\rho_{\phi}, \quad (11)$$

where in braneworld high energy limit ($\rho \gg \lambda$) $H^2 = \frac{4\pi}{3\lambda M_{PL}^2} (\rho_r + \rho_{\phi})^2$. Now extremization of the solution of the equation(11) gives extremum temperature during reheating $T_{ex}^{bh} = \sqrt[4]{\left\{ \frac{45\Gamma_{\phi} M_{\phi}^3}{8N^* \pi^3} \right\}}$. To use this idea in gravitino phenomenology we will start with gravitino Boltzmann equation

$$\frac{dn_{\tilde{G}}}{dt} + 3Hn_{\tilde{G}} = \langle \Sigma_{total} |v| \rangle n^2 - \frac{m_{\frac{3}{2}} n_{\tilde{G}}}{\langle E_{\frac{3}{2}} \rangle \tau_{\frac{3}{2}}}. \quad (12)$$

Hence solving equation(12) using $Y_{\tilde{G}} = \frac{n_{\tilde{G}}}{s}$ gravitino abundance at the end of reheating can be expressed as[7]:

$$\begin{aligned} Y_{\tilde{G}}^{brh}(T_f) &= \left(\frac{45\zeta^2(3)\tilde{\alpha}\sqrt{3\lambda}}{2\pi^7 M \Delta^2 N^*} \right) \left[\left(\frac{60\sqrt{\lambda}}{\pi N^* T_f} \right) \left(1 - \frac{T_f}{T^{brh}} \right) + \left(\frac{\pi (T_{ex}^{bh})^4}{32\Delta^2 T^{brh}} \right) \left(32 \left(\frac{T^{brh}}{T_{ex}^{bh}} \right)^4 - 1 \right)^{\frac{1}{4}} \right. \\ &\quad \left. \times \left\{ \frac{1}{\Gamma(\frac{3}{4})} + \frac{1}{\Gamma(-\frac{5}{4})} - \frac{2}{\Gamma(-\frac{1}{4})} \right\} \right]. \end{aligned} \quad (13)$$

which is plotted with respect to temperature in figure(1(b)).

5. Numerical estimations and data analysis

For a typical value of $C_4 \simeq D_4 = -0.7$, $56 < N < 70$ and decay width $\Gamma_{\phi} \simeq 2.9 \times 10^{-3} GeV$, mass of the inflaton $m_{\phi} \simeq 10^{13} GeV$ we have: $\Delta_s^2 \sim (1.440 - 3.126) \times 10^{-9}$, $\Delta_s^2 \sim 10^{-14}$, $n_s \sim (0.936 - 0.951)$, $n_t \sim 10^{-5}$, $r \sim (2.176 - 4.723) \times 10^{-5}$, $\alpha_s \sim -10^{-3}$, $\alpha_t \sim 10^{-6}$,

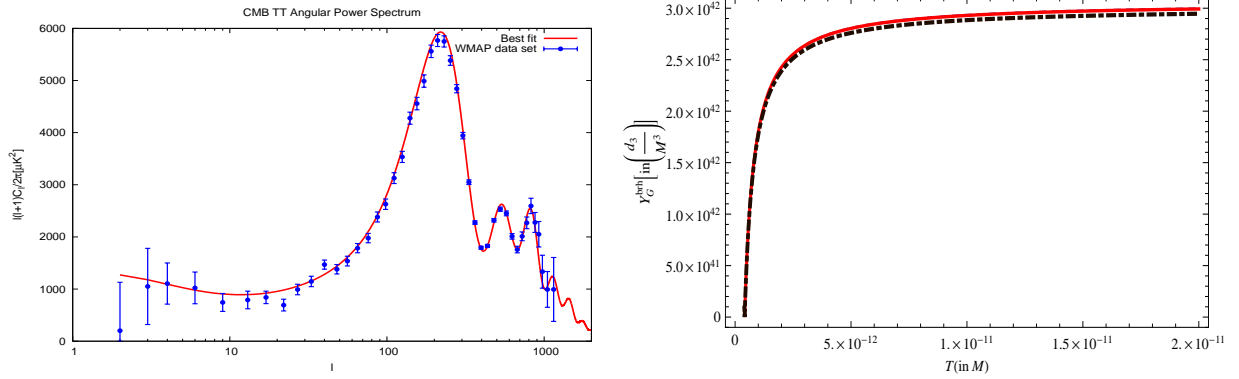


Figure 1. Variation of (a) CMB angular power spectrum C_l^{TT} with the multipoles l for scalar modes, (b) total gravitino abundance vs temperature

for boson $T_B^{brh} \simeq 7.6 \times 10^{10} GeV$, $T_{cB} \simeq 3.2 \times 10^{14} GeV$ for fermion $T_F^{brh} \simeq 7.8 \times 10^{10} GeV$, $T_{cF} \simeq 3.3 \times 10^{14} GeV$ and for mixture of species $T_M^{brh} \simeq 6.5 \times 10^{10} GeV$, $T_{cM} \simeq 2.8 \times 10^{14} GeV$, $T_{ex}^{bh} \simeq 7.0 \times 10^{10} GeV$, final temperature $T_f \simeq 10^6 GeV$, at the end of reheating $Y_{\tilde{G}}^{b-rad}(T_f) \simeq 2.1 \times 10^{-13} GeV^{-3} d_3$, where $d_3 = 6.594\tilde{\alpha} \times 10^{11} GeV^3$ and $\tilde{\alpha}$ is a dimensionless constant in MSSM. Now using CAMB [9] with the pivot scale $k_0 = 0.002 \text{ Mpc}^{-1}$, $H_0 \sim 71.0 km/sec/Mpc$, $\tau_{Reion} \sim 0.09$, $\Omega_b h^2 \sim 0.022$, $\Omega_c h^2 \sim 0.111$ and $T_{CMB} \sim 2.725 K$ we get $t_0 \sim 13.707 Gyr$, $z_{Reion} \sim 10.704$, $\Omega_m \sim 0.267$, $\Omega_\Lambda \sim 0.732$, $\Omega_k \sim 0$, $\eta_{Rec} \sim 285.10$ and $\eta_0 \sim 14345.1$.

6. Summary

In this article we have proposed a model of brane inflation derived from $N = 2, D = 5$ supergravity in the bulk leads to an effective $N = 1, D = 4$ supergravity in the brane including one-loop radiative corrections. Then the model has been employed in estimating the observable parameters, both analytically and numerically, leading to more precise estimation of the quantities using the publicly available code CAMB [9], which reveals consistency of our model with latest observations. Hence we have studied reheating and leptogenesis in brane by analyzing the reheating temperature, followed by analytical and numerical estimation of different phenomenological parameters showing deviations from GR results.

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